A Study of the Emittances of P1 during Proton Injections for Store 2070 and 2155

M. Church 1/16/2003

The following quantities for proton bunch 1 were extracted from SDA Viewer for the 36 Sets of the Inject Proton Case in stores 2070 (12/12/02) and 2155 (1/11/03). An earlier version of this report was presented at the 12/19/2002 Emittance Meeting.

T:SBDPSS – SBD rms bunch length for store 2070 *(It is not clear to this author if this quantity was actually the rms or the rms of a gaussian fit at the time of store 2070.)*
T:SBDPWS – SBD rms bunch length for store 2155
T:SBDPPS – SBD Δp/p *(The calculation of this quantity in the SBD front-end was changed by sqrt(2) between store 2070 and store 2155.)*
T:FWHPSG - sigma of E11 horizontal FW (1st and 2nd passes)
T:FWEPSG – sigma of E17 horizontal FW (1st and 2nd passes)
T:FWVPSG – sigma of E11 vertical FW (1st and 2nd passes)
T:FWHPFT – "goodness of fit" of E11 horizontal FW (1st and 2nd passes)
T:FWEPFT – "goodness of fit" of E17 horizontal FW (1st and 2nd passes)
T:FWVPFT – "goodness of fit" of E11 vertical FW (1st and 2nd passes)
(In this report, the "goodness of fit" parameters are all interpreted as being proportional to a chi-square.)
T :WHEP00 – FW horizontal emittance
T :WVEP00 – FW vertical emittance
T :WEEP00 – FW Δp/p

All SDA data appears to be good except the vertical FW emittance of Set 33 in store 2070 reports an error.

Recalculation of Δp/p:

For a bunch matched to a stationary RF bucket generated by a sinusoidal RF waveform, the equations of motion are

$$\frac{d}{dt}\left(\frac{\delta E}{h\omega_0}\right)=\frac{e\hat{V}\sin\phi}{2\pi h}, \quad \frac{d\phi}{dt}=\frac{-h^2\omega_0^2\eta}{E_0}\left(\frac{\delta E}{h\omega_0}\right)$$

where h = harmonic number, ω_0 = angular revolution frequency, δE = energy deviation from synchronous particle, $e\hat{V}$ = peak RF voltage, ϕ = phase with respect to RF (0 – 2π), E_0 = central energy, and η = slip factor. Assuming conservation of charge in 2-dimensional phase space, $\vec{\nabla}\bullet\rho\vec{V}=0$, gives a differential equation for the phase space density:

$$\frac{h^2\omega_0^2|\eta|}{E_0}\frac{\delta E}{h\omega_0}\frac{\partial\rho}{\partial\phi}+\frac{e\hat{V}\sin\phi}{2\pi h}\frac{\partial\rho}{\partial\left(\dfrac{\delta E}{h\omega_0}\right)}=0\,.$$

If the distribution is separable, $\rho=\rho_\phi(\phi)\rho_{\delta E}\left(\dfrac{\delta E}{h\omega_0}\right)$, then the solution is

$$\rho_{\delta E}\propto e^{-\frac{\delta E^2}{2\sigma_E^2}}\,,\quad \rho_\phi\propto e^{\frac{\cos(\delta\phi)}{\sigma_\phi^2}}$$

where $\delta\phi=\phi\text{-}\pi$ and $\sigma_\phi=\sqrt{\dfrac{2\pi h|\eta|}{E_0e\hat{V}}}\sigma_E$.

A monte carlo is used to generate the above gaussian energy distribution and almost-gaussian phase distribution, with the distribution truncated at the separatrix boundary. A polynomial fit is done to calculate Δp as a function of Δt. <u>In this case Δ refers to the rms value</u>. The fit is shown in Figure 1a. For $e\hat{V}$ = 1.05 MV at 150 GeV the result is

$$\Delta p=32.66\bullet\Delta t-3.527\bullet\Delta t^2+.1314\bullet\Delta t^3$$
$$A_{95\%}=-1.1235\bullet\Delta t+1.9281\bullet\Delta t^2-.49951\bullet\Delta t^3+.037512\bullet\Delta t^4$$

These expressions are also valid for 150 GeV beam in the Main Injector if the RF voltages are matched at transfer time. Δp is in units of MeV, and Δt is in units of nsec.

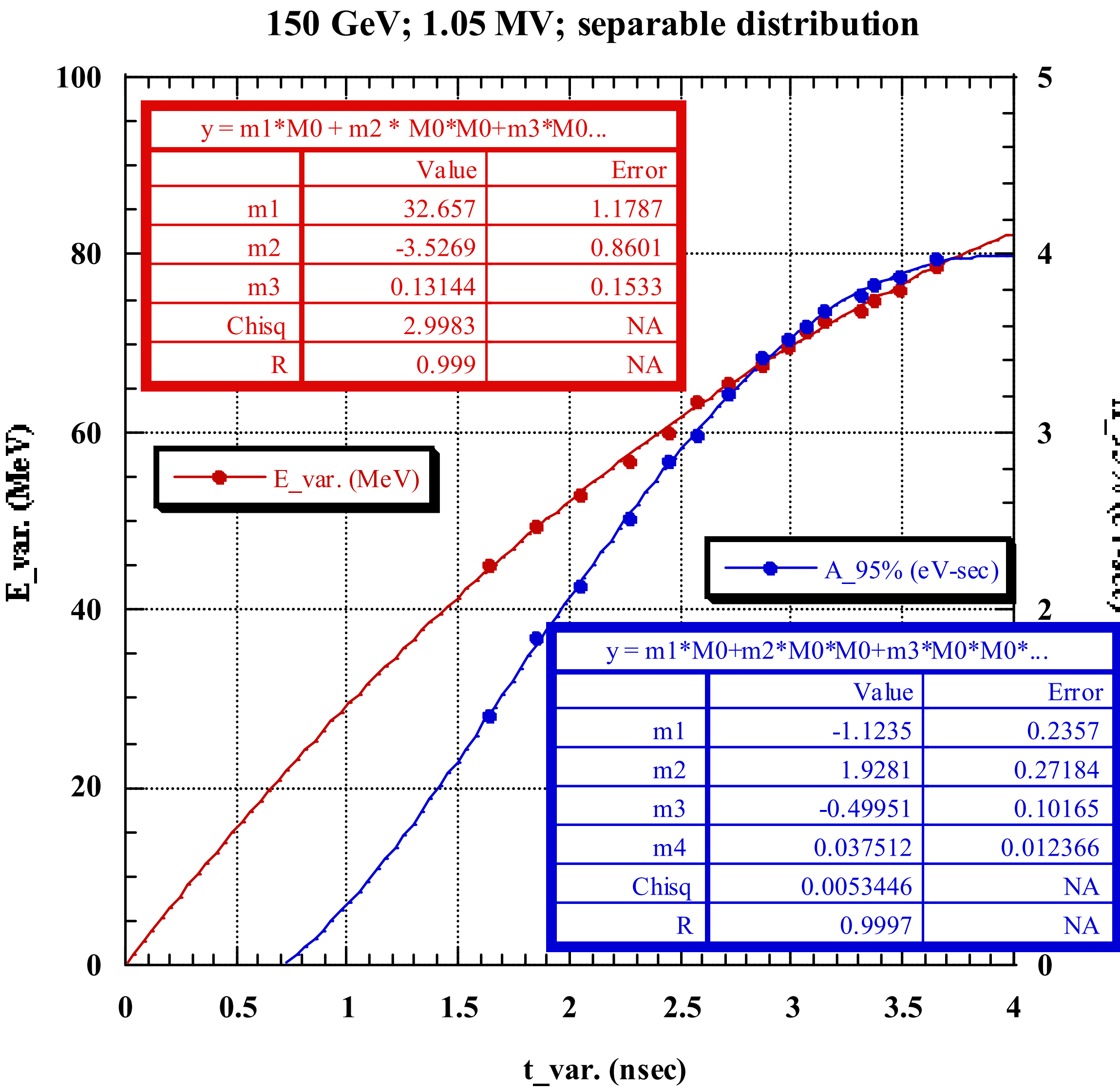

Figure 1a: Polynomial fits for momentum spread and longitudinal emittance @ 150 GeV

For reference, the fits for 980 GeV (low beta lattice) are shown in Figure 1b. For $e\hat{V}$ = 1.05 MV at 980 GeV the result is

$$\Delta p = 83.29 \bullet \Delta t - 10.562 \bullet \Delta t^2 + 0.8023 \bullet \Delta t^3$$
$$A_{95\%} = -.3628 \bullet \Delta t + 2.069 \bullet \Delta t^2 - .30744 \bullet \Delta t^3$$

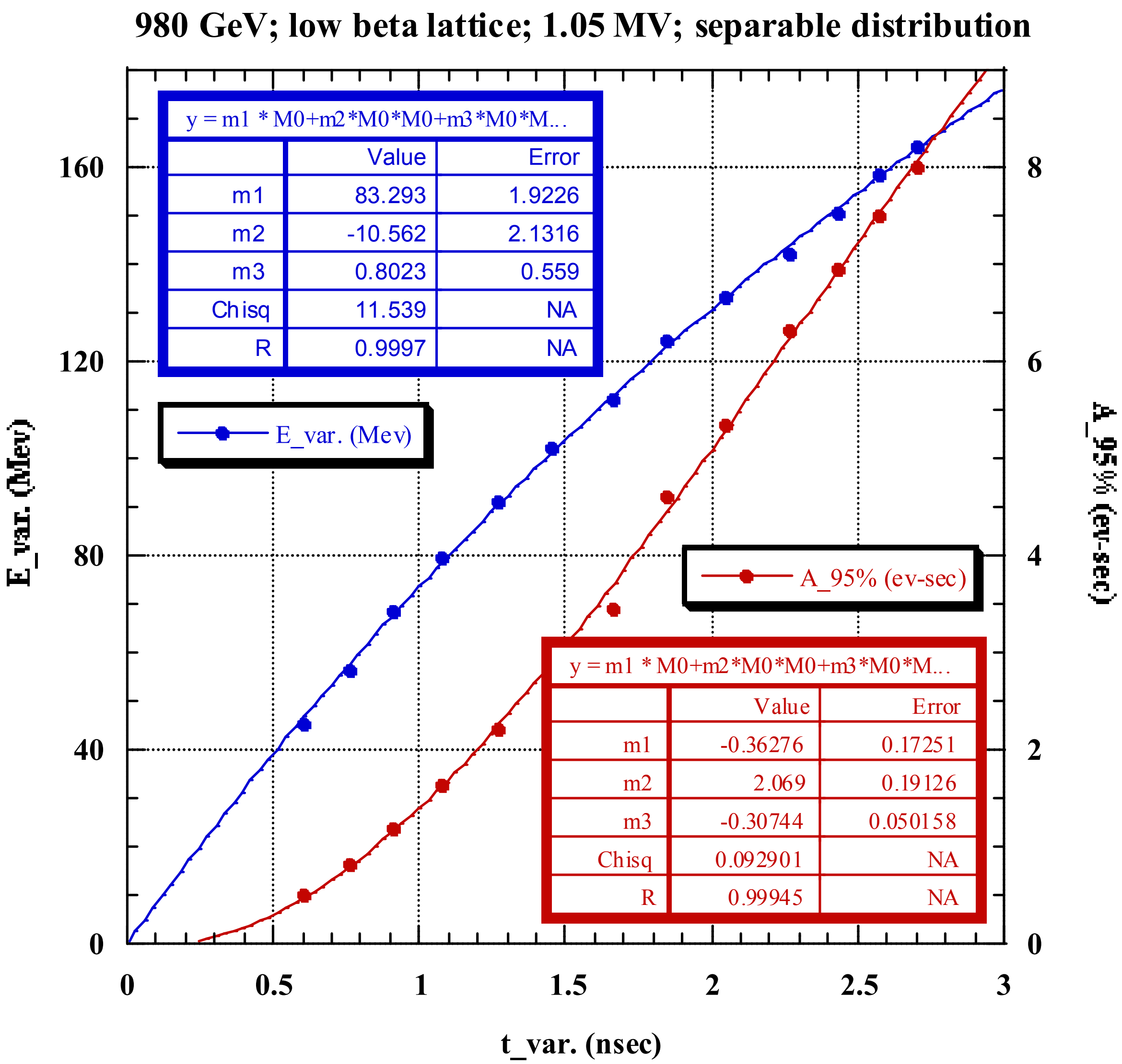

Figure 1b: Polynomial fits for momentum spread and longitudinal emittance @ 980 GeV

Linear least square fits for Δt and Δp/p as functions of time are shown in Figures 2a and 2b. Also shown are the SBD calculated values of Δp/p and the FW calculated values of Δp/p. RMS percent deviations from the linear fits are shown in Tables 1a and 1b.

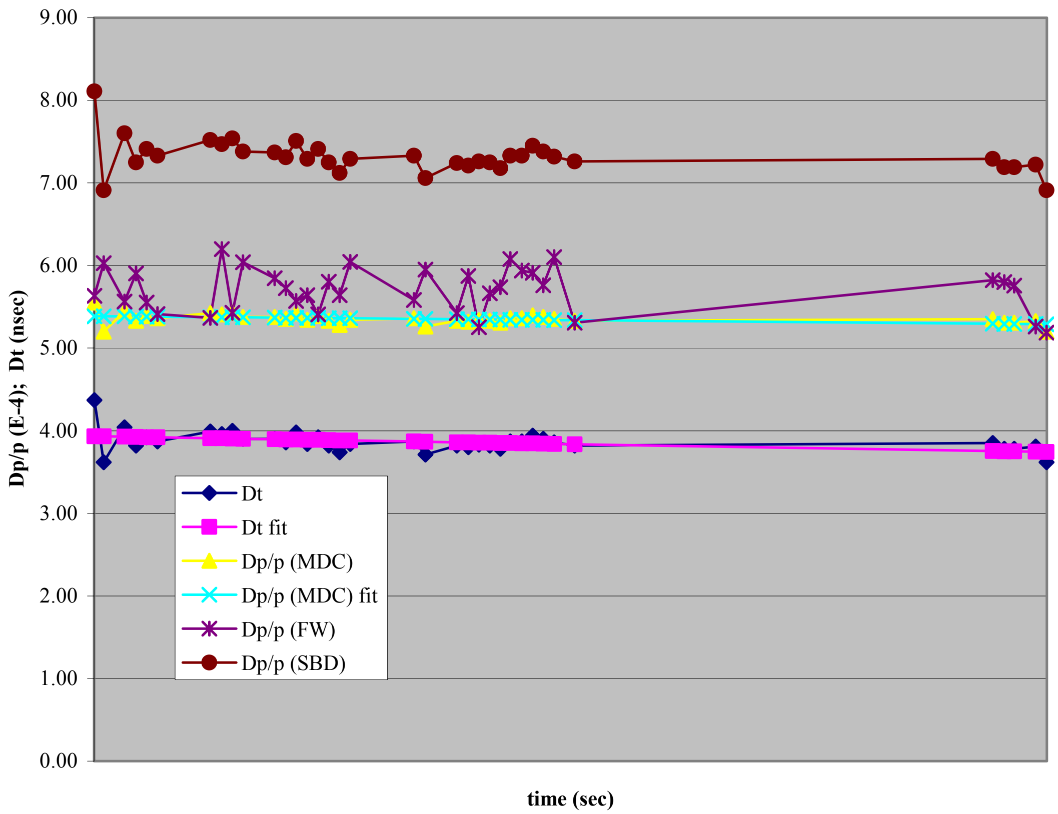

Figure 2a: Δt and Δp/p and fits for store 2070. Fits are not shown for the Δp/p values calculated by the SBD and FW. In this case, Δt from the SBD is T:SBDPSS.

	RMS deviation	RMS % deviation
SBD Δt (T:SBDPSS)	.112 nsec	2.9%
MDC Δp/p	.057	1.1%
SBD Δp/p	.180	2.5%
FW Δp/p	.261	4.6%

Table 1a: RMS deviations from straight line fit for Δt and Δp/p for store 2070.

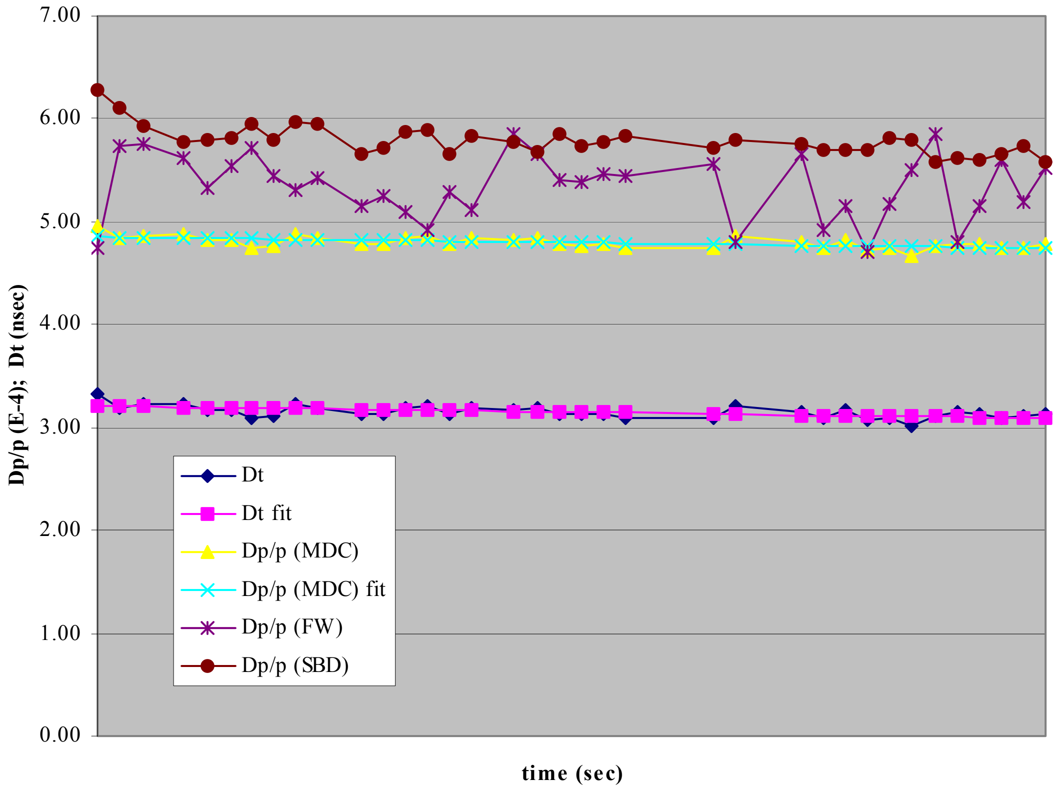

Figure 2b: Δt and Δp/p and fits for store 2155. Fits are not shown for the Δp/p values calculated by the SBD and FW. In this case, Δt from the SBD is T:SBDPWS.

	RMS deviation	RMS % deviation
SBD Δt (T:SBDPWS)	.046 nsec	1.5%
MDC Δp/p	.044	0.9%
SBD Δp/p	.104	1.8%
FW Δp/p	.309	5.8%

Table 1b: RMS deviations from straight line fit for Δt and Δp/p for store 2155.

<u>FW sigmas:</u>

The weighted averages of the 1st and 2nd pass FW sigmas are shown in Figures 3a and 3b. The weights are proportional to the inverse of the "goodness of fit" parameters. *(The author has found that better fits are obtained when the weights are reversed for the horizontal measurements. That is, T:FWHPSG[0] is associated with T:FWHPFT[36] and vice versa. For the vertical measurements, it is not clear which is the correct order. The author suspects there might be a front-end code error, or database error.)* Linear least square fits to a straight line vs. time are also shown. The rms deviations and percent deviations from the linear fits are shown in Tables 2a and 2b.

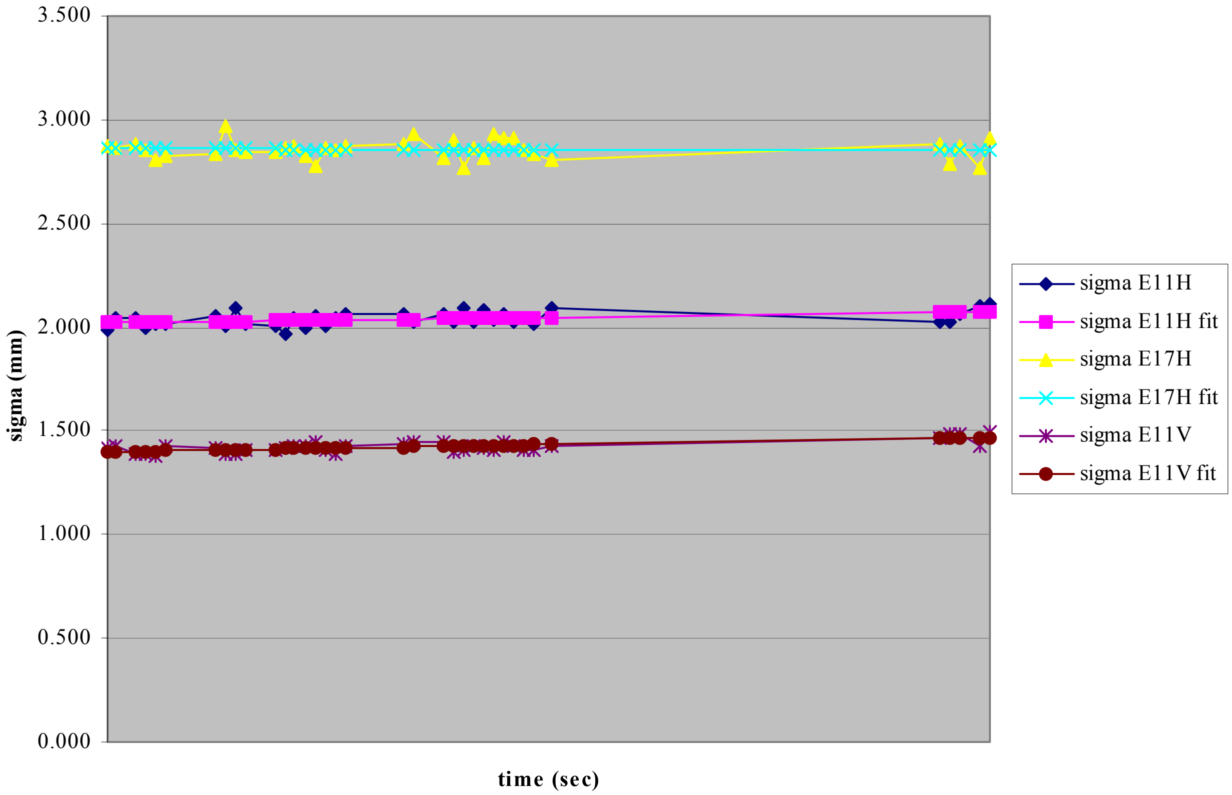

Figure 3a: FW sigmas and fits for store 2070. Data points are the weighted average of 1st and 2nd passes.

	RMS deviation (μm)	RMS % deviation
E11H	31	1.5%
E17H	47	1.6%
E11V	19	1.3%

Table 2a: RMS deviations from straight line fit for FW sigmas for store 2070.

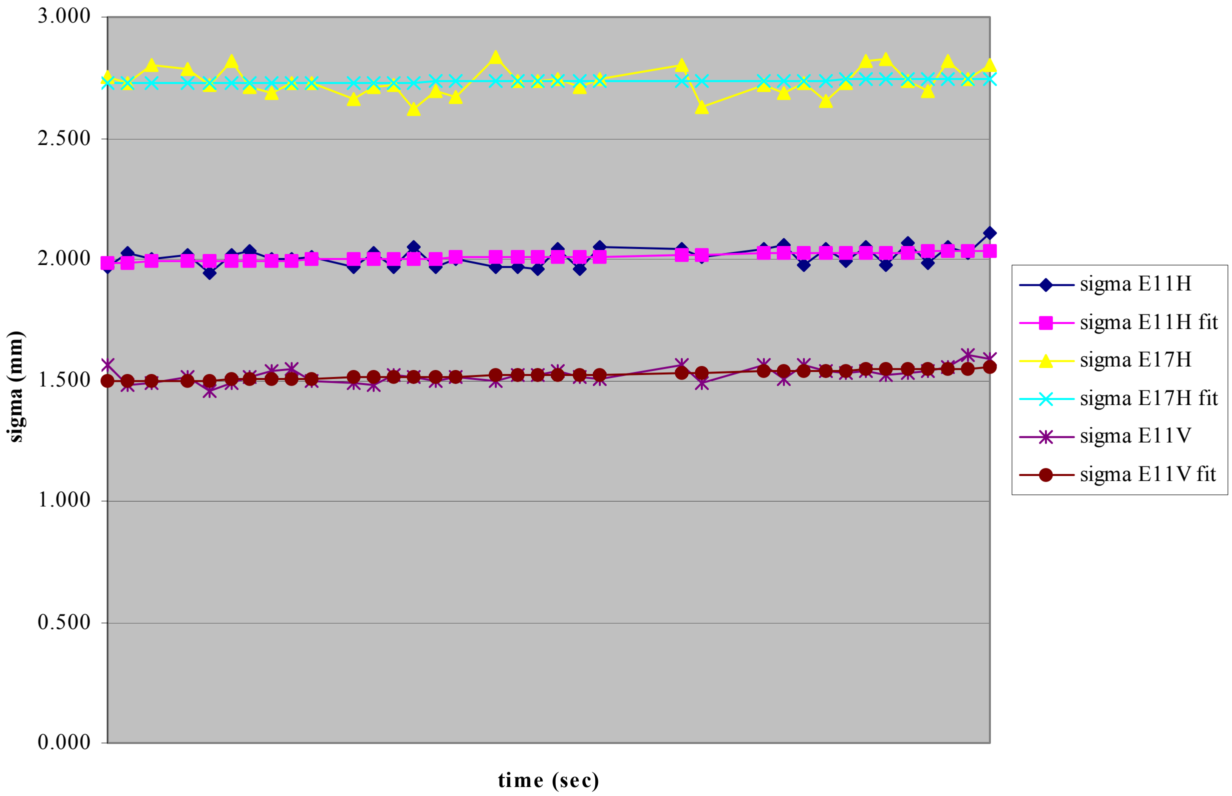

Figure 3b: FW sigmas and fits for store 2155. Data points are the weighted average of 1st and 2nd passes.

	RMS deviation (μm)	RMS % deviation
E11H	34	1.7%
E17H	43	1.5%
E11V	21	1.4%

Table 2b: RMS deviations from straight line fit for FW sigmas for store 2155.

Transverse emittance:

The emittance is calculated with the formula

$$\varepsilon_i = \left(\sigma_i^2 - \left(\frac{\Delta p}{p}\right)^2 D^2\right)\frac{6\gamma}{\beta} \qquad \text{[eq. 1]}$$

where i = E11H, E17H, or E11V; $\Delta p/p$ is from the fit from Figure 1; γ is the relativistic gamma; β and D are the lattice functions shown in Table 3.

	β (m)	D (m)
E11H	83.5	2.83
E17H	62.5	4.58
E11V	84.5	-0.09

Table 3: Lattice functions at FW @ 150 GeV as determined by V. Lebedev

Figures 4a and 4b show the FW horizontal emittance calculated with eq. 1. The emittance measured by the E17 wire should be the same as the emittance measured by the E11 wire. Possibly the formula is incorrect – it is valid if both transverse and momentum distributions are ~gaussian, and it is known that the momentum distribution at 150 GeV is not necessarily gaussian. Increasing the E17 β by 22% and decreasing the E11 β by 22% gives good agreement (see Figure 5), but it is generally agreed that the Tevatron β functions are known to better than 10% at 150 GeV [P. Bagley, V. Lebedev]. However, increasing the E17 dispersion function by 4% and decreasing the E11 dispersion function by 4% for store 2070 and 8% for store 2155 gives very good agreement between the two emittances (Figures 6a and 6b). The final horizontal emittance is calculated from the weighted sum of the E11 and E17 emittances (with unadjusted lattice functions). The weights are the inverse squares of the rms deviations from the straight line fits. The relative E11/E17 weights are .89/.11 for store 2070 and .89/.11 for store 2155 -- the E17 wire is hardly used at all. The final horizontal emittance and the FW front-end calculated emittance are plotted in Figures 7a and 7b. The vertical emittance (calculated with eq. 1) and the FW front-end calculated vertical emittance is plotted in Figures 8a and 8b. Note that the MI reported emittances at 150 GeV for P1 for store 2070 was 21.2 horizontal (bad!) and 26.6 vertical (very bad!), and the MI→Tevatron transfer efficiency was 89% (FBI narrow gate signals). Tables 4a and 4b show the rms deviations and rms % deviations for all these emittances.

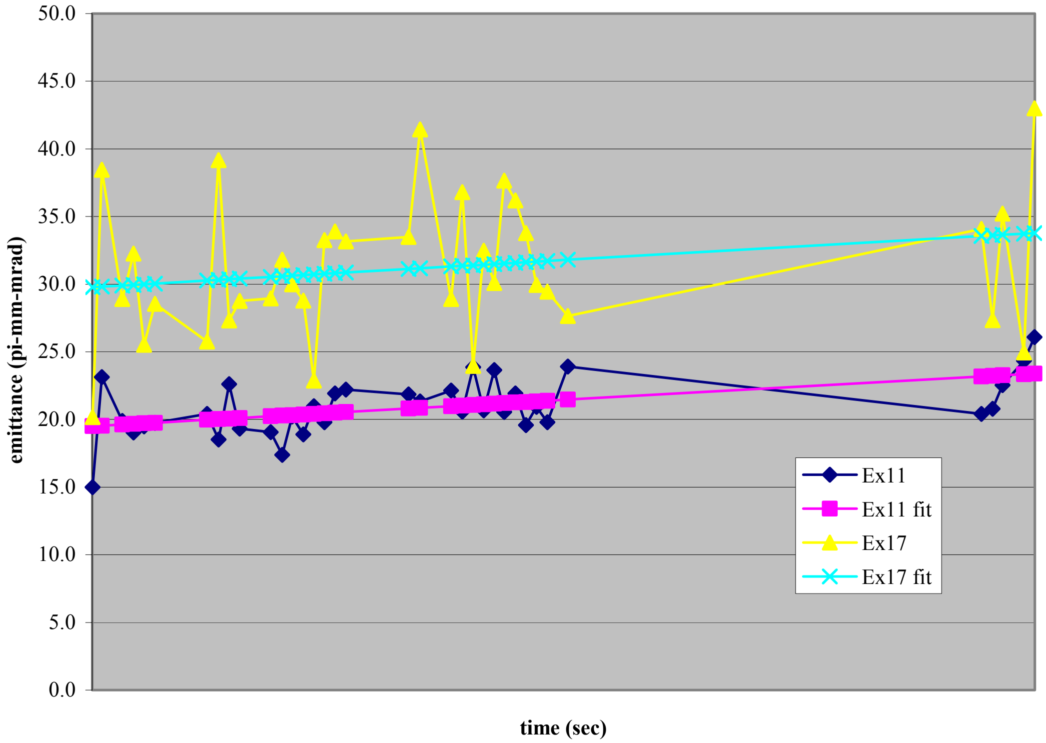

Figure 4a: FW horizontal emittances and fits with original lattice functions for store 2070.

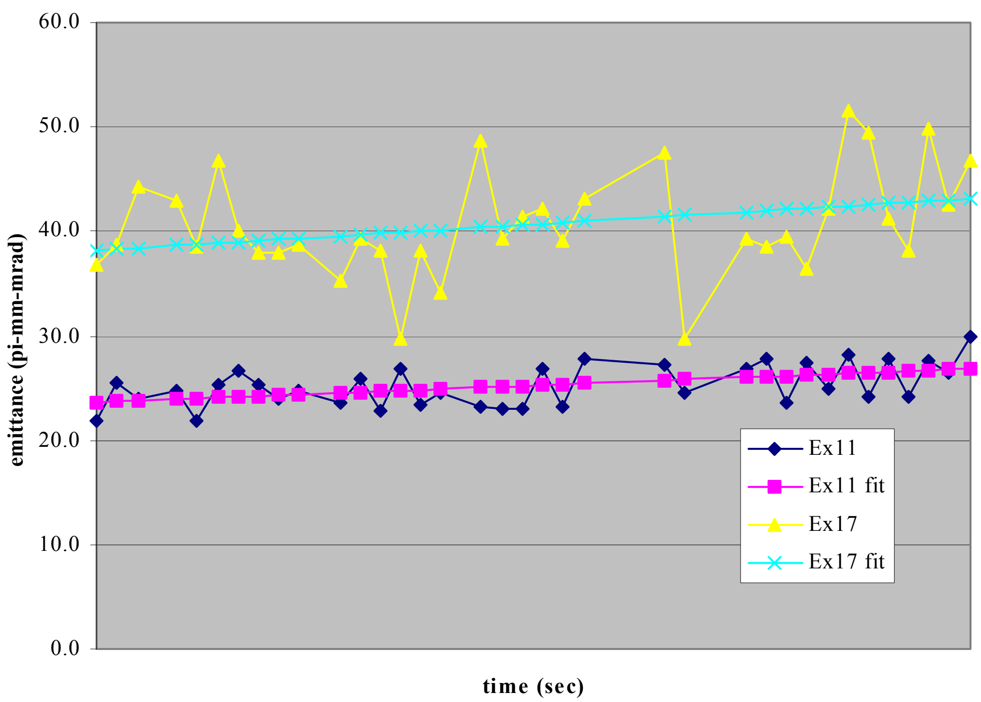

Figure 4b: FW horizontal emittances and fits with original lattice functions for store 2155.

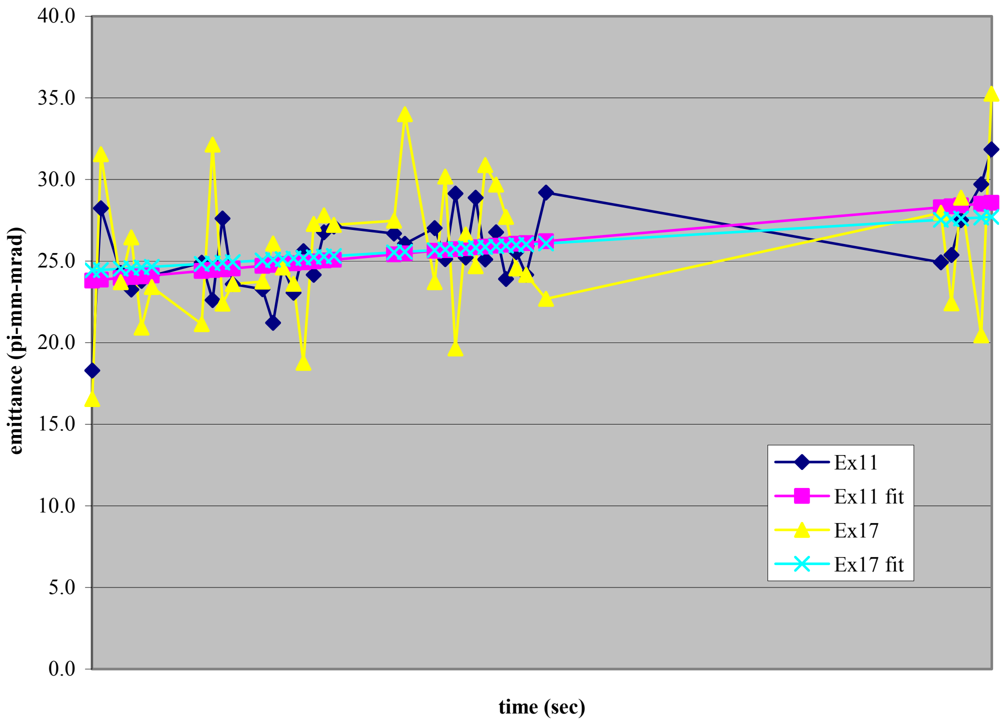

Figure 5: FW horizontal emittances and fits with β functions changed by 22% for store 2070.

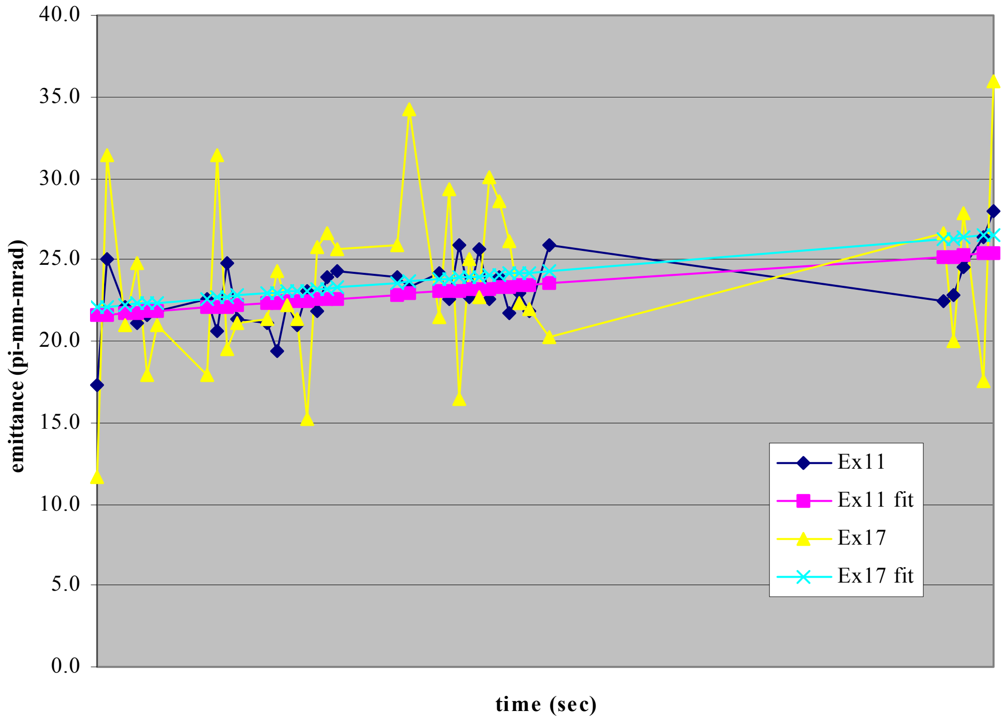

Figure 6a: FW horiz. emittances and fits with dispersion functions changed by 4% for store 2070.

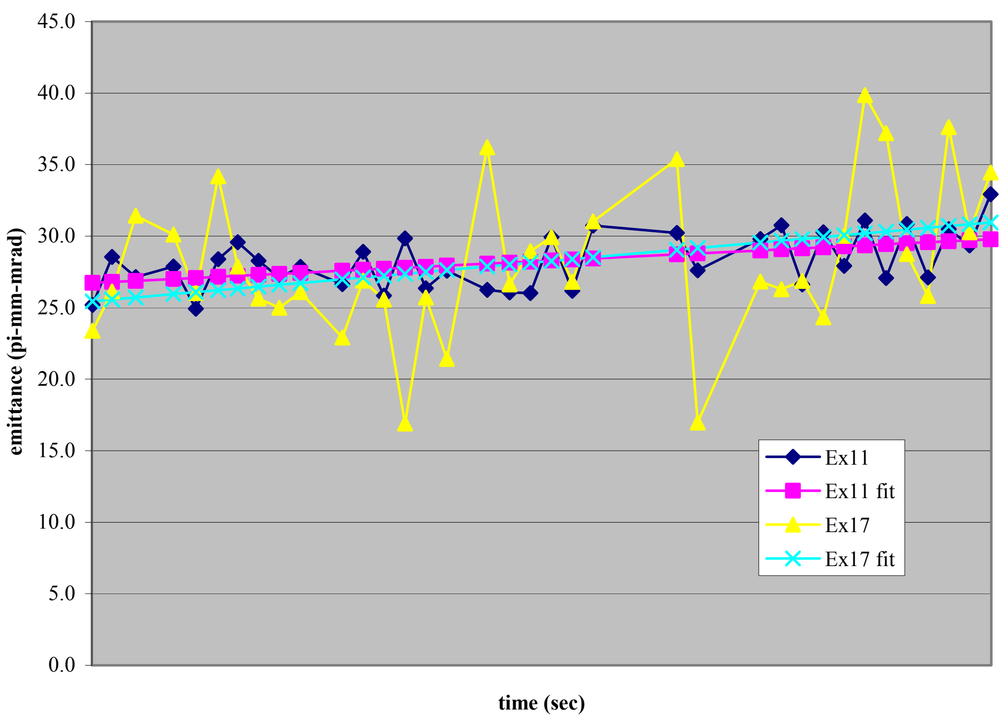

Figure 6b: FW horiz. emittances and fits with dispersion functions changed by 8% for store 2155.

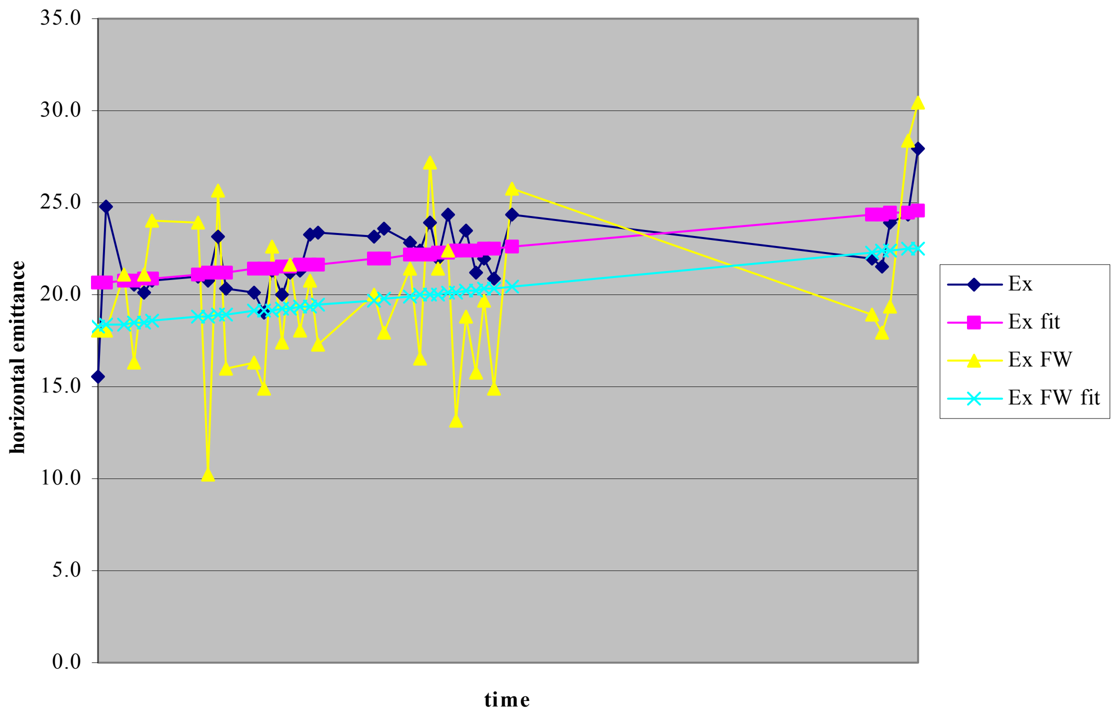

Figure 7a: MDC calculated and FW front-end calculated horiz. emittances and fits for store 2070.

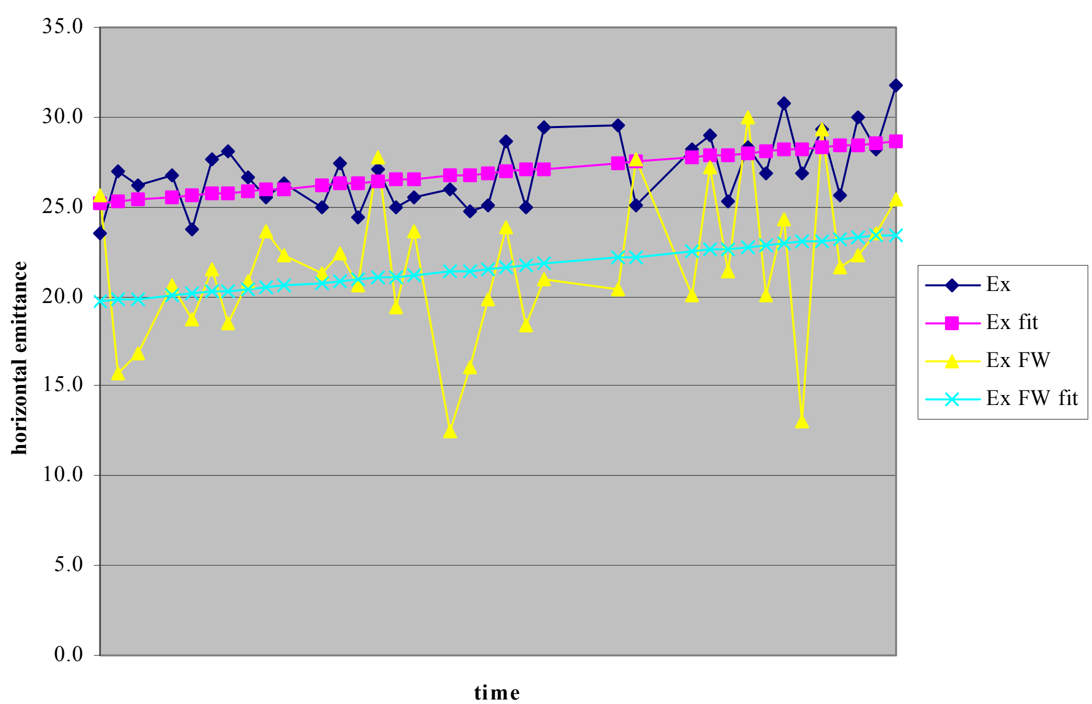

Figure 7b: MDC calculated and FW front-end calculated horiz. emittances and fits for store 2155.

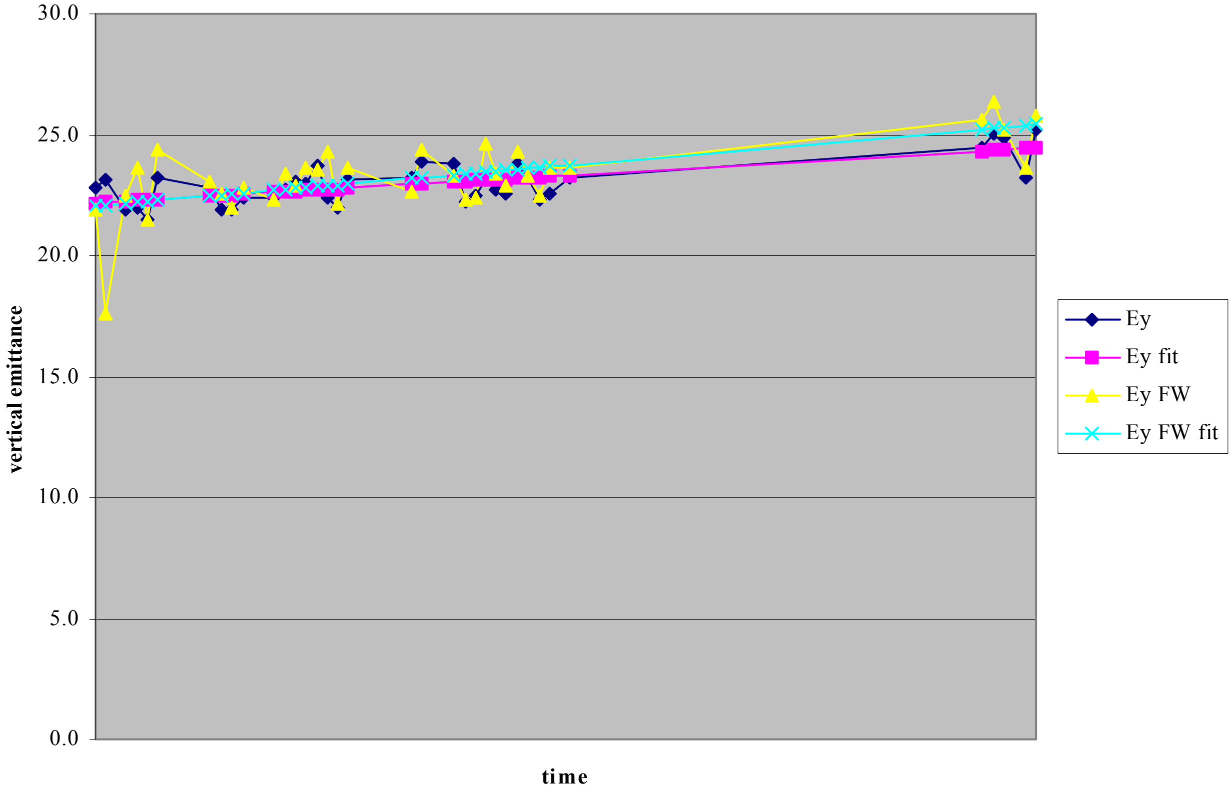

Figure 8a: MDC calculated and FW front-end calculated vertical emittances and fits for store 2070.

	rms deviation (π-mm-mrad)	% rms deviation	Average emittance (π-mm-mrad)
$\varepsilon 11_H$ [MDC]	1.8	8.5%	20.9
$\varepsilon 17_H$ [MDC]	4.9	16.0%	31.2
ε_H [MDC]	1.8	7.9%	22.0
ε_H [FW]	4.1	20.7%	19.8
ε_V [MDC]	0.6	2.7%	23.0
ε_V [FW]	1.1	4.8%	23.3

Table 4a: RMS deviations and % deviations from straight line fits for emittances for store 2070.

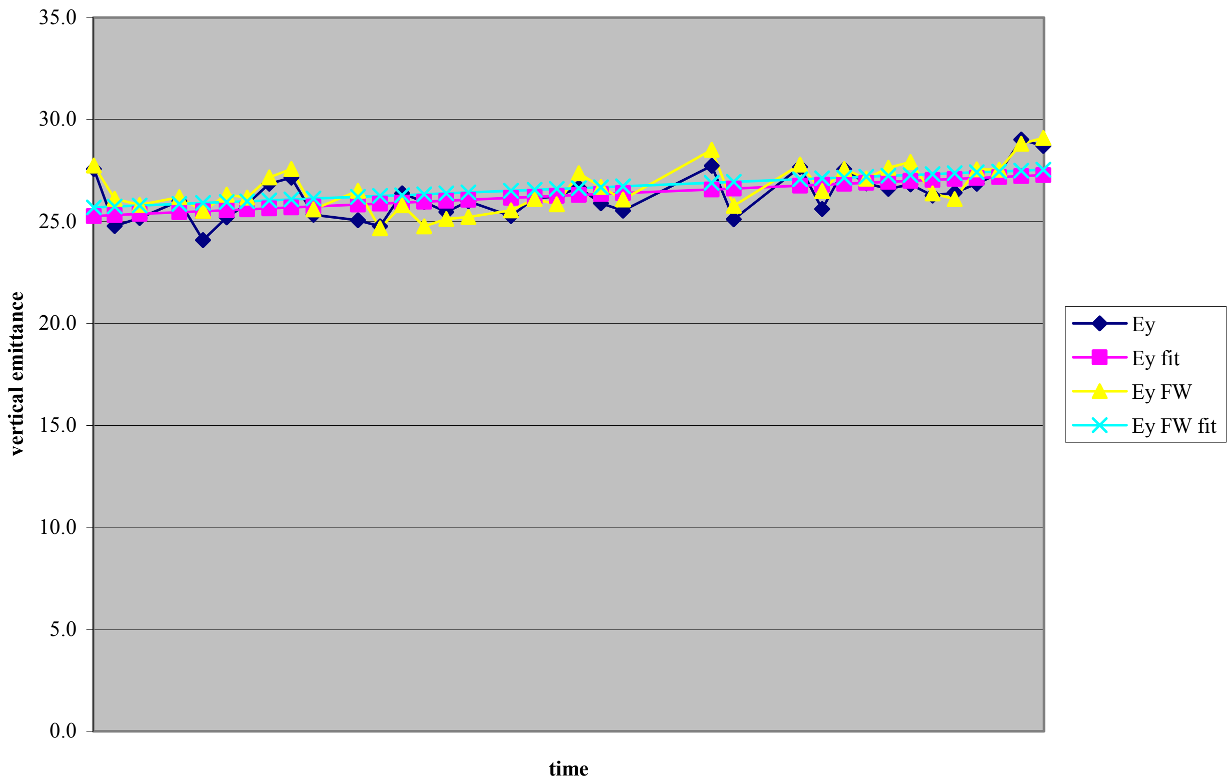

Figure 8b: MDC calculated and FW front-end calculated vertical emittances and fits for store 2155.

	rms deviation (π-mm-mrad)	% rms deviation	Average emittance (π-mm-mrad)
$\varepsilon 11_H$ [MDC]	1.7	6.7%	25.2
$\varepsilon 17_H$ [MDC]	4.8	11.8%	40.7
ε_H [MDC]	1.7	6.3%	26.9
ε_H [FW]	3.8	17.8%	21.6
ε_V [MDC]	0.9	3.4%	26.3
ε_V [FW]	0.9	3.5%	26.6

Table 4b: RMS deviations and % deviations from straight line fits for emittances for store 2155.